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There is such a point for all values of r *except* $r = -1$; and by taking all values of r we get all points on the line P_1P_2 *except* P_2 . These exceptional cases are avoided and we get simpler formulas, which are more easily derived, if we determine P such that $P_1P/P_1P_2 = r$ as follows; namely¹

$$x = x_1 + r \cdot \Delta x = x_1 + r(x_2 - x_1), \quad y = y_1 + r \cdot \Delta y = y_1 + r(y_2 - y_1).$$

To sum up, although marred by several inaccuracies in detail, the book as a whole is very good, thoroughly modern, and includes much in a small compass.

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The Algebra of Logic. By LOUIS COUTURAT. Authorized translation by LYDIA G. ROBINSON, with a preface by PHILIP E. B. JOURDAIN. The Open Court Publishing Co., Chicago, 1914. xiv + 98 pages. \$1.50.

This volume is a translation of volume number 24 in the Gauthier-Villars collection under the general title *Scientia*. These very useful manuals run about 100 pages each, in two series, the present number belonging to the physico-mathematical series. The translation has been well done although it is not exactly a literal rendering of the French style of sentence. A bibliography has been inserted and the preface presents, in brief form, a good history of the development of symbolic logic. These additions are distinct improvements and will make the book more useful.

The algebra of logic, as its name indicates, is a mathematical treatment of logic. It is not a new species of numerical calculation by symbols, but is a treatment of the field of logical notions by introducing symbols which have to be combined according to definite laws. It is an application of the methods of mathematics to logic, and is just as much applied mathematics as is mechanics. The only peculiarity in the situation is that in the reasoning processes employed, the symbols and laws relate to the reasoning process itself. The field of study in logic consists of classes of objects of the mind as marked by some distinguishing quality and the relations between such classes; assertions regarding mental objects and their relations; and relations themselves as objects with relations to each other. For example we may study the class of triangles that are distinguished as isosceles, and the class that are distinguished as having two equal angles. The identification of the two classes is done by logical processes. If we consider the problem of identification of classes of objects in general we are studying logic. Just as no one can get along in the world without using arithmetic consciously or unconsciously, so no one can get along without reasoning. If he becomes interested in the processes of arithmetic and the study of identities between numerical expressions in general, then he begins to study algebra. So too if he becomes interested in the process of reasoning he begins to study logic, and the most efficient way to do this is to apply the mathematical method; that is to say, to devise a system of symbols that will take the place of the mental objects under study, to parallel the mental combinations of these objects with

¹ These formulas are given, for example, by Ziwet and Hopkins, *Analytic Geometry*, p. 8.

combinations of the symbols, and to state as laws of operation the results of the mental transition from object to object.

Consider, for example, the process of reasoning called the syllogism. In its most simple form this consists in the succession of statements:

If objects A defined in any way have the quality B , and if objects that have the quality B must have the quality C , then the objects called A must have the quality C . We state this symbolically

$$(A < B)(B < C) < (A < C).$$

We are in a much better position to discuss the syllogism from this visualization of it in symbols than if we try to discuss it in words and in sentences. We can indeed reduce the manipulation of our logical formulæ to a few very simple rules which not only enable us to produce the usual results of logic, but we can solve very much more difficult questions. For examples see Jevons' *Principles of Science*, or his *Studies in Deductive Logic*.

We can do more however, for if A , B , C above are not simple classes but are propositions, we have a formula relating them in precisely the same manner as the syllogism above, and all our formulæ become susceptible of another interpretation. We can thus reduce our two-fold logic in the main to a single treatment, which is a distinct gain. The author does this throughout the book. The formulæ developed are interesting in showing what can be done in the discussion of general questions. However it should be remembered that for any concrete case, the fundamental rules of the symbolism are all one needs to solve the problem.

A word is in place perhaps as to why one should study symbolic logic at all. The best answer is that it is desirable for precisely the reasons that one should study algebra, or theory of groups. While everyone possibly is gifted with correct reasoning power, at least so long as the problem is simple, it is often not possible to carry an intricate set of premises in the mind and deduce the correct conclusion. Nor is it possible easily to solve the reverse cases and say, for example, what would be the simplest set of hypotheses that would make certain propositions follow as consequences. With the machinery of symbolic logic this becomes quite manageable.

As an example we mention the famous problem of Venn stated thus:

The members of a board are either bondholders or shareholders, but not both; all the bondholders are on the board. What conclusions can be drawn?

As an example of inverse logical problems we may quote the following: What are the premises that enable us to conclude that objects with the quality C do not have the quality E , and that objects without the quality C must have the quality E , do not have the quality A , and also either do not have the quality B or else have the quality B but do not have the quality D ?

These simple problems illustrate the character of work that can be handled easily by symbolic logic but not easily in the ordinary way. They exemplify in their solution also the fact that the syllogism is only one of many ways of

reasoning. This however we have not space to enter into, and it should be followed up after reading the book before us, in some of the more extended treatises mentioned in the bibliography in this book. Every student of mathematics ought to read through some book on logic, and the present one is an admirable introduction to the subject.

JAMES BYRNIE SHAW.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND R. P. BAKER.

PROBLEMS FOR SOLUTION.

Note.—An additional supply of good live problems is desired, especially in algebra, calculus, and mechanics.—EDITORS.

ALGEBRA.

When this issue was made up, solutions had been received for numbers 424–427

428. Proposed by FRANK IRWIN, University of California.

If the roots of the equation

$$x^n - na_1x^{n-1} + \binom{n}{2}a_2x^{n-2} + \dots = 0$$

are all real, the condition that they should all be equal is $a_1^2 = a_2$. A proof of the sufficiency of the condition is readily obtained from a consideration of derivatives. A proof is desired not based on such considerations.

429. Proposed by C. N. SCHMALL, New York City.

It is given that d_1, d_2, d_3 , are the greatest common divisors of y and z , z and x , x and y , respectively; also that m_1, m_2, m_3 , are the least common multiples of the same pairs of numbers. If d and m are the greatest common divisor and least common multiple, respectively, of x, y , and z , show that

$$\frac{m}{d} = \left(\frac{m_1 m_2 m_3}{d_1 d_2 d_3} \right)^{1/2}.$$

430. Proposed by V. M. SPUNAR, Chicago, Illinois.

Solve the equations algebraically and also graphically: $x^y + y^x = xy$, $x^x + y^y = x + y$.

GEOMETRY.

When this issue was made up, solutions had been received for numbers 452–454

457. Proposed by NATHAN ALTSHILLER, University of Washington.

AB and AC are respectively a diameter and a chord of a circle whose center is O . The lines joining B to the extremities of the diameter perpendicular to AC , meet AC in the points M, N . Express the angle MON in terms of the angle CAB .

458. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Given edges l, m , and n of a parallelepiped and the angles a, b , and c which the edges make with one another. Show that, if $s = (a + b + c)/2$, the volume equals

$$2lmn \sqrt{\sin s \sin (s - a) \sin (s - b) \sin (s - c)}.$$